Chapter 2 – Local operation: mechanisms and operators

Series and cycles

Now that we have posited the boundaries of the formalism that allows us an initial approach to the musical space, prior to a finer structuring, it may seem anachronous to come back to conventional serial operators to commence the description of certain useful syntax tools.

All the more so since, in the works of Webern for example, they may have seemed to be sufficient in themselves to determine this space and even in contradiction with any other pre-determination. In fact, the notion of order between a finite set of sound objects may be in opposition to the free play of the subsets of these objects.

However, this contradiction is only apparent; first of all, the set of objects handled by the serial operators is only a totality with the exception of the congruence (modulo); in addition, the vertical projection of a section of the discourse generally determines a *de facto* selection, with limited local control. It is the random nature of these global spatial relations that emphasised the weakness of the serial language, throwing it into crisis.

Let us return to the real functions of the series:

- Perform the functional statistical distribution of notes that escape the tonal order.

- Deduce this operator distribution, able to organise the totality of audible sounds, within a quantified scale, from an order of succession that is as restricted as possible

- Attach to this order of succession, and to its transformations by the previously defined operators, a structural function.

The first condition, that tends to use the serial mechanism as a pseudo-random “note generator”, was pushed to its limits by Jean Barraqué [5.13], who defined a sequence of series, based on a given series considered as a permutation of degree 12, by applying the order of terms of this permutation to itself as many times as possible, either by falling back onto the initial permutation (cycle), or by arriving on permutation 0 which displaces no objects. We can see that this mechanism, which brings the most abstract properties of the permutation groups into play, meets the second condition, but is resolutely removed from the third. What is more, such a treatment, which leads to an organised proliferation, is typically the domain of computer calculation.

The second condition, which intends to institute an order of objects in a totality, initially only concerned the objects themselves (notes) and not their relations (intervals). The organisation of the totality of intervals within a series, already touched on by Hiller and Isaacson [6.1], will be developed in what follows. The third condition implies an analysis of the internal symmetries that can exist within the series itself. We will also give some examples of this.

What remains significant, in my opinion, in the serial approach is the definition of abstract operators [5.1] adapted to computer processing, to which we shall return.

The computer to the rescue: calculating balanced cycles.

We will not repeat the chronology of the emergence of the Schoenberg series here. What was specific about the approach was, in my opinion, facing up to the evanescence of the tonal hierarchies, to substitute a notion of order applied to a set of notes modulo-12, to the notion of hierarchised selection typical of the tonal sphere. Schoenberg himself described the system as a "method of composing with 12 notes which are related only to one another". [1.14].

It would be useful to reread the analysis of symmetries internal to Webern series and the general problem of the internal structure of a series as conceived by Pierre Boulez [5.5]. In his writings we find, among other things, the observation of a particularity of the initial series of the *Suite Lyrique* by Alban Berg (fig 2.2), the fact that it includes all the possible intervals, the second half of the series being the exact mirror of the first, transposed to the diminished fifth.

It would not be precise to say that my own investigations in this direction started from this observation. At the time (1958-1960) I was looking for a more radical organisation, which could give an irrefutable meaning to what Schoenberg wanted to do (relations of the notes to one another). To do this, it was necessary that a series not be characterised only by an order of elements of the whole, which means an order of the totality of intervals with respect to a reference note, but also to a continually varied order of the proximity between these notes, that is to say to a series of intervals. On the one hand, I experimented by conceiving a work based on a series of intervals which used only one subset of $\mathbb{Z}_{12}$, and therefore had the character of a mode. "*Dualités pour violon et piano"* (1962) is based entirely on this ambiguity. Such a transient conception constituted the exact complement to Schoenberg's approach (Series of pitches: totality of elements of $\mathbb{Z}_{12}$ in general using only one subset of proximity intervals. Series of intervals: totality of intervals modulo-12 using only one subset of the elements of $\mathbb{Z}_{12}$).

From this point, the tracks were laid down for reaching the limit: finding series which are both series of pitches and at the same time series of intervals. I endeavoured to find such series, and realised that their synthesis was not straightforward. In fact, as a result of the corresponding dual constraint, while it was easy to begin, the gradual exclusion of the remaining pitches and intervals was most often impossible to achieve, and it took me several hours to discover one, whose internal structure did not necessarily seem to me to be satisfactory. In fact, it was a question of tracing a graph in a vast tree structure whose permitted paths decreased at each stage. My professional contacts with computing (since 1961 I had been in charge of a hybrid calculation laboratory at the Euraton research centre in Ispra, Italy) provided me with a solution: having a Fortran programme written by a computer programmer/analyst, A. Debroux, who, following my instructions, was able to calculate all of the series with this dual characteristic. [6.2]

At the same time I was able to observe that a balanced series was closed by constitution, that is to say the 12^{th} interval closed the cycle on the first note.

Definitions

Let $\mathbb{Z}_{12}=(0,1\ldots.11)$ be the set of 12 notes in the equal tempered octave.

$\mathbb{Z}_{12}$ forms a group in relation to the previously defined addition.

Consider all the ways of forming an ordered sequence of 12 elements with all or some of the elements in the group. This will be set £ of the arrangements of 12 objects 12 to 12 with repetitions that form a symmetrical group S (£) with the cardinal 12^{12}

or A_0 = < $a_0, a_1, \ldots a_{11}$ > one of these sequences $a_k \in \mathbb{Z}_{12}$ k = 0,1, ... 11

and $d_i = a_{i+1} + a_i^{-1}$ the distance between two successive elements which we will call an interval.

Reciprocally, we have:

$$a_i = a_{i-1} + d_{i-1} = a_0 + \sum_{i=0}^{i-1} d_i$$

lastly $\delta_i = a_i + a_0^{-1} = \sum_{i=0}^{i-1} d_i$ represents the interval between element i and element 0 or variance.

If $a_0 = 0$, the sequence of δ_i coincides with that of a_i

Now let

D_0 = < $d_{0,} d-1, \ldots d-11$ > the ordered sequence of intervals of A_0

And suppose that

$a_{i+1} \neq a_i$ whatever the value of i (no unison between two successive notes).

In this case, the $d_i \in \Delta_{12}$ with $\Delta_{12} = \{-6, -5, -4, \ldots -1, 1, \ldots 6\}$

if A_0 is such that $a_i \neq a_j$ whatever the values of i and j, we say that A_0 is a series of pitches.

The set $E \in \pounds$ of A_0 is formed of all the permutations of the elements of $\mathbb{Z}_{12}$, which forms a symmetrical group of cardinals $12! = 479.001.600$

If the suite D_0 associated with A_0 is a permutation of Δ_{12} , we say that A_0 is an interval cycle

The interval cycle is closed on itself (the 12th interval is the distance between a_{11} and a_0)

Demonstration

We have $a_1 = a_0 + d_0$

$a_2 = a_1 + d_1$

....

$a_{12} = a_{11} + d_{11}$ a_{12} being the element defined by

$$a_{12} + \sum_{i=1}^{11} a_i = a_0 + \sum_{i=1}^{11} a_i + \sum_{i=0}^{11} d_i$$

that is to say $a_{12} + a_0^{-1} = \sum_{i=0}^{11} d_i$

but $\sum_{i=0}^{11} d_i \equiv 0$ (mod 12)

hence $a_{12} \equiv a_0$ cqfd

let E' be the set of interval cycles. $E' \in £$ as interval 6 coincides with its inverse, the symmetrical group E' of interval cycles will have the cardinal $\frac{12!}{2}$

the intersection $L = E' \cap E$ $L \subset £$ represents the set of ordered suites A_0 which have the double property of being series of pitches and interval cycles. We say that these suites are <u>balanced cycles</u>. Following $L \subset E'$, they have the property of closure that was previously demonstrated.

Back to computer processing:

The initial programme provided, after reduction to $a_0 = 0$ 1928 BC (balanced cycles) (as the irony of numbers would have it, this is the year of my birth. Patrick Greussay told me he rewrote the programme and recalculated the number of balanced cycles).

In fact, a later version of the programme that I wrote used the property of closure, excluding all the cycles that coincide with the recurrence, the inversion and the recurrence of inversion (in the Schoenbergien sense) of the previously calculated cycles, analysed in closed loops.

A secondary analysis of the programme highlights those cycles of which a form, deduced by the above operators and the “dephasing” operator, to be described later, coincides with the cycle itself (50 cycles).

To simplify the classification of the material obtained, a successive programme submitted all of the balanced cycles for analysis by subsets of 3 to 8 notes, underlining

the major and minor chords

the various seventh chords

the suites of 5 to 7 notes belonging to a major or minor tone (including the modes obtained by inversion of the minor. It is easy to observe that all the other chords or modes analysed keep their properties by inversion)

the suites of 5 to 8 notes belonging of residue classes mod 12

the suites of 5 to 8 notes belonging one of the modes of limited transposition the most commonly used by Olivier Messiaen. (M_2^k with k = 1, 2, 3 see above).

This analysis was able to show:

38 BC coinciding with their recurrence

6 BC with double coincidence

35 BC comprising no classified chords

51 BC comprising no classified chords other than a 7^{th} diminished

(residue classes 3_i) etc...

The series already mentioned from Berg's *Suite Lyrique* was one of the self-coincident symmetrical BCs.

After these various filtering operations I kept 65 BC, which seem best able to meet the structural criteria I set myself.

We can therefore observe that balanced cycles are a "rare" commodity (final forms: $70 x 12$ transpositions

Possible series: $12!$ hence a probability of $\frac{70\times 12}{479\times 10^6}\approx 1{,}6\times 10^{-6}$) and, without computers, even if I had devoted all my time to the task, I would still not have finished the exploration today.

Formalisation of operators on the BC

The $A_0 = (a_0, D_0)$ pair is also used to characterise a BC

a_0 is the 1^{st} term, or origin.

Operator V (inversion)

$VA_0 = < v_0, v_1, \ldots v_{11} > = (a_0, VD_0)$

with $v_k = a_0 - \sum_0^{k-i} d_i$ (we use the notation $m_i^{-1} = -m_i$)

$VD_0 = < -d_0, -d_1, \ldots, -d_{11} >$

Operator R (recurrence)

$RA_0 = < r_0, r_1, \ldots r_{11} > = (a_{0,} RD_0)$

with $r_k = a_{(-k)}$

$RD_0 = < -d_{11}, -d_{10}, \ldots, -d_0 >$

Operator T (transposition)

$T^j_{A_0} = < t_0, t_1, \ldots t_{11} > = (a_0 + j, D_0)$

with $t_k = a_k + j$ $j = 0, 1, \ldots 11$

Operator Φ (transposition)

$\Phi A_0 = < \varphi_0, \varphi_1, \ldots \varphi_{11} >$

with $\varphi = a_{k+1}$

Some relations

Where I is the identity operator $IA_0 = A_0$

We have V.V = I

R.R = I

$T^i T^j = T^{(i+j)}$ $T^{12} = I$

$\Phi^i \Phi^m = \Phi^{(i+m)}$ $\Phi^{12} = I$

Composition of operators

We have $R.V = V.R$

$V.T^J = T^J.V$

$R.T^j = T^j.R$

$V.\Phi^k = \Phi^{\text{¿}} k.V$

($\Phi_d^k . R = R . \Phi_d^{(-k)}$ Φ_d operator Φ applied to D_0)

$R . \Phi^j = \Phi^{(-j)} . R$

$T^r \Phi^j = \Phi^j . T^k$ demonstration in the appendix.

The operators V, R and T are commutative among each other. They therefore generate 4 sub-groups:

$T^j A_0, T^j V A_0, T^j R A_0, T^j RVA_0 \quad j=0,1,\dots 11$

or, all of the 48 terms of the Schoenberg operations.

Φ commutates with V and T; it does not commutate with R but the univalent correspondence $R\Phi^j = \Phi^{-j} R$ makes it possible to render explicit the whole set of terms generated in each sub-group by the cyclic permutations Φ^i

The general term of vocabulary V or U^x , will be characterised by 4 indices:

U^x (i, j, α, β) = $\Phi^i T^j V^\alpha R^\beta A_0$ with $\begin{matrix} i=0,1,\dots 11 & \alpha=0,1 \\ j=0,1,\dots 11 & \beta=0,1 \end{matrix}$

V will therefore comprise, in general, 576 terms.

Special cases

(fig 2.5)

if in D_0 there is $d_i = -d_{(i+6)}$ the word A_0 is 'V-symmetrical'

if in D_0 there is $d_i = -d_{(2-i+1)}$ the word A_0 is 'R-symmetrical'

In both cases, the corresponding vocabularies only have 288 terms.

Internal structure of a series or cycle

Pierre Boulez described certain properties inherent to the structure of a series.

Barraqué also studied series related to the foundation of Alban Berg's *Suite Lyrique*, and detailed the analysis of *Allegro Misterioso* and of Webern's variations for piano.

However, it could be useful to repeat the exercise, applying to a BC, in order to indicate that the updating of symmetries, if there are any, are specific to a BC and that such an analysis should be done for each new BC. Let us take for example BC No. 357 used in "Perspectives", my String Quartet with multiple solutions. If we take the precaution of highlighting the even and odd intervals in two separate circular modulo-12 representations, with their orientation, the symmetries appear clearly.

First we observe that the odd intervals include the semitones, major thirds and tritones (form A) while the even intervals include tones, major thirds and fourths (form B). This being said, and by extension with the properties of the groups, we can highlight the main transformations using the conventional operators T, R and V.

On form A

1°) Transpositions (rotations)

- a rotation of π (T^6 or tritone) conserves the disposition relative to odd intervals but retrogrades it.

- a rotation of $\frac{\pi}{2}$ (T^3 or minor third) exchanges the intervals within a group of 4 notes

t (do - do#, sol - fa#) R (do - fa #, do #- sol)

R (la - mi *b*, ti *b* - mi) t (mi *b* - mi, si *b* - la)

m (fa - re, la *b* - if) m (la *b* - fa, ti – re)

a rotation of $\frac{3\pi}{2}$ (T^9 or major sixth) operates a similar exchange:

t (do - do#, sol - fa#) R (fa # - do , sol - do #)

R (la - mi *b*, ti *b* - mi) t (mi – mi *b*, la - ti *b*)

m (fa - re, la *b* - ti) m (fa - la *b*, re – ti)

2°) Inversions (symmetries in relation to diameters)

- symmetry in relation to diameter $\frac{\pi}{12}$ does not modify the minor thirds; retrogrades the others.

- symmetry in relation to diameter $\frac{7\pi}{12}$ retrogrades the minor thirds; does not change the others.

On form B:

1°) Inversions

A symmetry in relation to diameter $\frac{5\pi}{12}$ does not modify even intervals.

To summarise, there are three main types of transformation

The rotations of k_1 ($\frac{\pi}{2}$ + $\frac{\lambda\pi}{12}$) , k_1 = 0,1,2,3 (3 families)

The symmetries in relation to k_2 ($\frac{\pi}{12}$ + $\frac{\lambda\pi}{2}$) (6 families)

The symmetries in relation to ($\frac{5\pi}{12}$ + $\frac{\lambda\pi}{12}$) (12 families)

The first two operate exchanges of intervals on the same pairs of notes.

The third leaves even intervals stable.

Thus, networks can be built on subsets of possible forms, linked through close relations between families of intervals.

Related forms

Different BCs exist, certain subsets of which remain identical (fig 2.8). Another, wider possibility consists in finding the closest series of pitches and series of intervals from a given BC through successive exchanges of symmetrical intervals in the BC (t ($t \rightleftarrows' \bar{t}$ $T \rightleftarrows' \bar{T}$, etc). At the time, I described a sub-programme for the main programme to calculate and classify the BC, which gives these families automatically.

General modes of sequencing of series and cycles

Here, again, Boulez provided indications [5.5] about the types of sequencing used in his own compositions.

Using the properties of the BC, I defined junction operators allowing the constitution of strings of different lengths, as the exploration of possible strings for a BC can be automated, allowing them to be grouped into subsets. (A specialised software programme called MANUCYCLES has since been devised by M. Mesnage to calculate all the possible subsets that match a pre-determined mode of sequencing (concatenation).

For this we use a generalisation of the use of the conventional operators (R,V, R-V) on existing strings.

“Junction” operations or concatenation with coincidence

To build the strings of terms, we define junction operators.

- 1- Weak junction: or two terms B_0 and C_0

The junction is made by $c_0 = b_{11}$ (coincidence of notes)

- 2- First order junction: or two terms $B_0 = (b_0, D_0)$ and $C_0 = (c_0, G_0)$

  The junction is made by coincidences of intervals, that is to say $g_0 = d_{11}$

- 3- Second order junction

  with the same notations $g_0 = d_{10}$

If the 1st and 2nd order junctions are made alternatively between elements of two sub-groups, the resulting strings have a univalent form and are closed.

Algorithms in music

The necessities of computer processing have familiarised us with the automatic symbol generation mechanisms known as algorithms (the word algorithm shares a root with the word algebra, and was defined by the French scientific language consultative council as: “a system of symbols and the operating rules relative to these symbols”), mechanisms formalised by the theory of finite automata. However, the actual concept they describe has always been present in music, starting with its trivial form, repetition.

Every pupil in the musical composition class knows the harmonic sequence, with or without modulation, and here we shall give only a brief reminder of three examples, taken from keyboard literature for convenience.

a- non modulating sequence taken from the 9th of 25 sonatas for harpsichord by Domenico Scarlatti: established on a descending path of the scale of A minor in its melodic form, it is a precursor of the sequences of parallel chords used by Debussy.

b- non modulating sequence taken from the Rondo in Beethoven’s Sonata for piano op 2 N03 in C major.

c- modulating sequence taken from the Rondo in Beethoven’s sonata for piano op 28 in D major.

These are ascending or descending translations of motifs built on simple harmonic relations establishing the elementary step which can then be reproduced automatically, where necessary with re-initialisation (leaps of 10ths every 4 groups of set b), with a view to a path that is significant in itself.

This path, if it is long enough (examples a and b) is easily memorised and can therefore be predicted by the receiver, who is sensitised either to its changes in orientation or to its completion.

Note an important difference between the two types of harmonic sequences:

> the modulating sequence precisely respects the relations of the initial motif in its translations; its "geometric" proportions are fixed and the sequence is a suite of applications of the motif in $\mathbb{Z}_{12}$.
>
> The non modulating sequence, on the other hand, brings into play an "arithmetical" operation, as the basic motif is not constituted by the intervals themselves but by the number of elements it runs through.

The example b is in fact built on an ascending third "idea of a motif" (a concept dear to Jean Barraqué), that is to say, in the notation of the chapter on scales, a suite of applications of the motif (i, i+2) in the subset P_3^M (scale of E *b* major) of $\mathbb{Z}_{12}$.

Also note that example b is a compromise between the two types of mechanisms, for the other constituent of the motif, the one which precedes the third, is a semitone higher irrespective of the membership of the initial note; the translations of this will therefore belong not to P_3^M but to $\mathbb{Z}_{12}$. There are abundant examples of mechanisms of this type, in particular in the scores of J.S. Bach, to the extent that we could talk about a style that is "*déterministe par morceaux*" in numerous sections of his work.

However, here we will try to illustrate a series of mechanisms of the same order encountered in the composition of Olivier Messiaen, who made frequent use of it.

An almost totally algorithmic piece: *"l'Echange"*

Taken from *20 regards sur l'Enfant-Jésus* for Piano by Olivier Messiaen. Except for the last 7 bars, the whole piece can be described from a block of duration of 11 quavers (2 bars of the score) comprising 4 separate groups. These 4 groups have dates of occurrence, durations and a fixed number of constituents within the period. This is therefore a simple case of operating in a single loop.

The 1st group, formed of 2 sub-groups of density 2 and 3 is totally repetitive (pitches, articulations), only the intensity is constantly increasing.

The other 3 groups operate on identical algorithms

$n_{i+1}=n_i,\ n_{i+1}=n_i+1,\ n_{i+1}=n_i-1$ (i number of the order of passage in the loop)

applied to isolated elements or to successions of elements.

A more detailed formalisation is not necessary to grasp the principle. Note that the limit chosen by Messiaen (12 increments), avoided the reprise of harmonic ratios with the exception of the octave.

Similarly, the last bars, which operate first by

> - truncating the loop (reduced to 4 quavers by elimination of the last 7) and anticipation of group G_2 by one unit (mes.25 to 27) and 3 repetitions;

- further truncation by elimination this time of the first two (mes.28) and 3 repetitions leading to silence, beyond which an abrupt dilation in time breaks the loop, and concludes by a fixation outside the algorithm.

The same algorithm is found in the typical form of a 5-quaver period loop in a section of piece X (p.62-63 of the score); the loop operates on 13 periods.

Same process in the strette of the Fugue (Regard VI p 35.38 of the score) in a canon for 3 voices; again, loop on 12 periods, followed by a variant for two real voices, one of the voices (the bass) being identical, except for the octave, to the leading voice of the previous example re-initialised (period: 15 semiquavers), the other is the subject in an opposite movement with extension by note repetition (period: 23 semiquavers).

Lastly, the same algorithm in Regard XX, in an 11-quaver loop, present in 3 sections (left hand) with re-initialisation every 4 periods and increase in density (2 notes in the octave, 3 notes) then of the number of events (quaver, quaver, then semiquaver in beats).

This algorithm is contrapuntal with a motif of 12 sounds of equal value repeated twice and of fixed position in the period. The motif has only 4 different forms, which are repeated on each re-initialisation of the algorithm. No obvious algorithm appears between these 4 forms.

An exercise of algorithmic extrapolation

This is therefore what we are now going to turn our attention to, by trying to answer the question: is there a more general algorithm of which these 4 forms are only a subset?

I am not suggesting this is the approach followed by Messiaen; I am convinced it is not, but it is instructive, given the restricted number of liberties he left himself, to solve this problem with a view to a wider application of algorithmic mechanisms.

Let us observe, therefore, the 4 groups in question; as the pitches of the chromatic total are in invariable position, this can be represented numerically modulo-12 (b). For the first 5 notes, the mechanism is evident. In the following 7, a single regularity is equivalent to ($si \rightarrow 11$). However, between groups 1 and 2 we observe the usage for these 7 notes of the dephasing operator Φ (cyclic permutation of n terms from a position on the left) at power 3. Note in passing that this same operator at power 2 can describe the displacement of terms 2 to 4. We can therefore imagine a <u>branched</u> algorithmic sequence, that is to say passing from one odd group to the next by applying an algorithm to be defined, while the even groups will be deduced from the previous odd group by applying 2 local operators.

Let *A* be a sequence of n terms.

We will call local dephasing, noted $\Phi_1(k, l; A)$ the cyclic permutation of *l* terms of *A*, beginning with the *ki th*, of a term on the left $(k+1-1)$.

In particular, a permutation of two neighbouring terms is noted $\Phi_e(k, 2; A)$

Let us call *A(12)* the 1st group in fig 2.18a

We move from the 1st to the second by applying operators

$\Phi_e^2(2,3; A)$ and $\Phi_e^4(6,7; A)$ (fig 2.19a)

Now let us find the algorithm for passing from the 1st to the 3rd group. It is suggested by the invariant succession 560; we then apply:

$$A'_0 = \Phi_e(7,6; A_1)$$

$$A''_0 = \Phi_e(10,3; A'_1)$$

$$A_3 = \Phi_e(9,2; A''_1)$$

As for group 4, according to our hypothesis of branched sequencing, it is deduced from a form of odd group by operations a; find this form: we will deduce group x from it.

We still have to pass from the last existing group, A_3 to this group x;

We apply the same type of operators:

$$A'_3 = \Phi_e^2(9,4; A_3)$$

$$A_x = \Phi_e(10,2; A'_3)$$

By defining the global algorithms

$A_{2i+2} = F(A_{2i+1}) = \Phi_e^2(2,3; A_{2i+1}) \oplus \Phi_e^4(6,7; A_{2i+1})$ (we can only perform the summation of operators to the extent that their areas of actions are not connected)

$$A_{4i+3} = X(A_{4i+1}) = \Phi_e\Big[9,2\Big[\Phi_e\Big[10,3\Big[\Phi(7,6; A_{4i+1})\Big]\Big]\Big]\Big]$$

$$A_{4i+3} = Y(A_{4i+1}) = \Phi_e\Big[10,2\Big[\Phi_e^2(9,4; A_{4i+3})\Big]\Big]$$

we could define the production sequence of the A_i from A_1

The string of operators $X \rightarrow Y$ gives 8 distinct forms, we will therefore calculate the 16 A_i of which we will find the musical transcription.

As for the read algorithm that respects the starting sequence $(A_1 - A_2 - A_3 - A_6)$, we can find a possible form which analyses, in 3 successive passages, the 16 A_i with a repetition of each group; its total length before reprise is therefore 32 groups, whereas Messiaen uses 7 x 4 = 28. We could find others, related, for example, to the evolution of algorithms of the left hand.

This exercise is useful for situating the degree of literal repetitivity tolerated in a language of this type. In fact, in his presentation, Messiaen already repeats each group twice $A_1 - A_2$, $A_2 - A_2$, etc. which is 14 repetitions in all.

Other procedures of Messiaen based on durations.

He analysed them himself [5.2]

To recap:

- The canon of durations on fixed pitches from a given voice, based on a sequence of irregular rhythmic variations of a cell (Regard XIV – 3 voices)

- On the <u>same rhythmic sequence</u>, imitation by augmentation 3/2 (or "addition of a point"). The "given" voice is constructed harmonically on the mode $M_6^3 = 2_0 \cup 6_1$ (see formalisation of modes) at constant density 4, and its imitation in mode $M_4^4 = 3_2 \cup 6_3 \cup 6_4$ at constant density 4. They serve as a *décor* to the "thème de Dieu" deployed in Regard I, constructed first of all in mode $M_2^1 = 3_0 \cup 3_1$ then in $M_2^2 = 3_1 \cup 3_2$

as $M_6^3 = 6_0 \cup 6_1 \cup 6_2 \cup 6_4$

$M_4^4 = 6_2 \cup 6_3 \cup 6_4 \cup 6_5$

$M_2^1 = 6_0 \cup 6_1 \cup 6_3 \cup 6_4$

$M_2^2 = 6_1 \cup 6_2 \cup 6_4 \cup 6_5$

their intersections are easily obtained:

$M_6^3 \cap M_4^4 = 6_2 \cup 6_4$

$M_2^1 \cap M_6^3 = 6_0 \cup 6_1 \cup 6_4$ $M_2^2 \cap M_6^3 = 6_1 \cup 6_2 \cup 6_4$

$M_2^1 \cap M_4^4 = 6_3 \cup 6_4$ $M_2^2 \cap M_4^4 = 6_2 \cup 6_4 \cup 6_5$

$M_2^1 \cap M_6^3 \cap M_4^4 = 6_4$ $M_2^2 \cap M_6^3 \cap M_4^4 = 6_2 \cup 6_4$

The chords employed in the leading voice and its imitation are not directly related to the rhythmic canon: nonetheless, there is a liaison to the extent that for each voice, the cycle of chords corresponds to the cycle of 17 basic durations, with an additional internal loop.

In fact, there are 10 basic chords in the leading voice:

$m_1 m_2 m_3 m_4 m_5 m_6 m_7 m_8 m_9 m_{10}$

distributed thus:

$\underline{m_1 m_2 m_3 m_4 m_5 m_6}$ $\underline{m_1 m_2 m_3 m_4 m_5 m_6}$ $\underline{m_7 m_6}$ $\underline{m_8 m_9 m_{10}}$

and 9 basic chords in the imitation:

$P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9$

distributed thus

$\underline{P_1 P_2 P_3 P_4 P_5 P_6}\ \underline{P_1 P_2 P_3 P_4 P_5 P_6}\ \underline{P_7 P_1 P_2}\ \underline{P_8 P_9}$

each of these 3 presentations of the canon are identical, but initialised differently (7th quaver, 3rd quaver, 1st quaver) in relation to the theme they accompany.

Finally, elementary algorithms of decreasing and then increasing durations on a fixed chord ($d_i = d_{i-1} - 1, d_i = d_{i-1} + 1$ between 1 and 16 x) at the beginning and end of Regard XVI (page 122 score), and decreasing durations on ascending chromatic translation of a chord, superimposed on a symmetrical increase on a descending chromatic translation of the same chord at the beginning of Regard XVIII (page 138 score), with precise retrograding of the set at the end of the same piece.

Profile actualisation algorithms

In chapter 1 we explained the notion of profile, of pitches and durations. How can these profiles be applied in the course of the language to predefined scales?

I'll give an example taken from *Transe Calme* for piano (Riotte 1974). This is an organisation based on the encounter of 3 "voices" C_1 , C_2 , C_3 associated with a principal monody.

To each of these voices a specific scale is attached; the relations between these scales are explained in another chapter. For the moment, note that they constitute respectively n_1 , n_2 and n_3 notes. We choose a number N prime with n_1 , n_2 and n_3 which corresponds to a sampling of N successive sounds from each scale.

As the sounds are successive, each sample will be entirely determined by the choice of the lowest sound (or the highest) in the sample.

The algorithm for reading these samples in blocks could be defined, for example, by alternating upward and downward slides of a sound.

Fig 2.25 gives the principle of the mechanism and the sequence of the corresponding limit-sounds for voices C_3 . We see that it corresponds to a progressive restriction of the ambitus, hence an indication of the possibility of controlling the areas covered by a wise choice of algorithms.

Each sample, corresponding in the example to a set of 17 notes, is divided into 4 groups of notes (3, 6, 4, 4) starting from the lowest note, each group being assigned to a particular organisation predominantly monodic, rhythmic and harmonic (m, r, h). Let g_1, g_2, g_3, g_4 be these groups of notes. They will be presented in sequence in the order g_3, g_2, (g_1+g_4) with the respective functions m, r and h.

We will give the example of organisation of the function m, which introduces the new notion of cyclic sampling from a node.

A node is a sequence of N memorised abstract events that constitute an infinite loop (that is to say, that the node constituted is modulo-N), which we will call upon partially for an effective actualisation.

Each abstract event of the node could be a k-uple, that is to say, a complex formed by a relative situation of pitch, duration, articulation and dynamic. Fig 2.27a shows the node attached to the organisation m in the example given.

If we take successive samples from this node, we obtain distinct events that nonetheless all come from a single predefined organisation.

In fact, let node N ($e_1, e_2, \cdots e_n$) e_i be event i

and sample $N(e_i ; k)$ e_i 1st term of the sample, k number of terms of the sample

We only need apply, for example, an algorithm such as

$$e_i \rightarrow e_{i-1}$$

to obtain n different possible actualisations of k terms. We stress the general nature of such a concept as node sampling; all the figures obtained, for instance, in fig 2.27b do not presage the sounds to which they can be applied; these depend on the chosen scale.